Assignment 1. Solution.

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Q1. The figure shows a delayed ramp function. The slope of the ramp $= \frac{b}{(a+b)-a} = 1$. Thus the function is a unit-ramp, delayed from $t=0$ by 'a'.

$$\therefore F(s) = \int_0^a 0 \cdot e^{-st}\,dt + \int_a^\infty (t-a)e^{-st}\,dt$$

$$= \int_a^\infty (t-a)e^{-st}\,dt \qquad \text{(The first term is zero.)}$$

Let $t-a=\tau \quad \therefore \; t=\tau+a$ and $dt = d\tau$

$$F(s) = \int_0^\infty \tau\, e^{-s(\tau+a)}\,d\tau$$

$$= e^{-sa}\left[\tau \frac{e^{-s\tau}}{-s}\Bigg|_0^\infty - \int_0^\infty \frac{e^{-s\tau}}{-s}\,d\tau \right]$$

This term evaluates to zero.
(Hint: Use L'Hospital rule)

$$= e^{-sa}\left[-\int_0^\infty \frac{e^{-s\tau}}{-s}\,d\tau \right]$$

$$= e^{-sa}\left[-\frac{e^{-s\tau}}{s^2} \right]_0^\infty$$

$$= e^{-sa}\frac{1}{s^2} = \frac{e^{-sa}}{s^2}$$